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An investigation of inertial one-phase flow in homogeneous model porous media

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Abstract. *Our interest in this work is the stationary one-phase Newtonian flow in a class of homogeneous porous media at large enough flow rates requiring the introduction of the inertial forces at the pore-scale. At the macroscale, this implies a nonlinear correction to Darcy's law i.e. a nonlinear relationship between the filtration velocity and the pressure gradient. The objective here is to analyze the nonlinear correction on some periodic models of porous media with respect to the Reynolds number and the pressure gradient orientation relative to the principal axes of the periodic unit cell. Our results show that, in the general case, for ordered structures, the inertial correction to the Darcy's law, i) involves a non-symmetric tensor even if the structure is isotropic in the Darcy regime (i.e. is characterized by a spherical permeability tensor); ii) is neither aligned with the applied pressure gradient nor with the mean flow indicating that the macroscopic force exerted on the structure is not a pure drag; iii) the onset of the deviation from Darcy's law is characterized by a correction which varies with the mean velocity magnitude to the cube (weak inertia regime); iv) the quadratic correction, classically referred to as the Forchheimer correction, is an approximation. This approximation does not hold at all for certain particular orientations of the pressure gradient in this type of structure.*

1 Introduction

Our interest in this work is focused on one-phase flow in homogeneous porous media when the flow rate is high enough for inertia to be taken into account. Typically, this implies a correction to Darcy's law that has been the subject of a great deal of work over the past decades [1, 2]. Most of the time, this correction is analyzed, both numerically and experimentally, through a scalar term corresponding to the modulus of the filtration velocity. In this work, we investigate the correction in its complete -vectorial- form with the help of numerical simulations performed on model 2D periodic porous structures.

Our results are interpreted on the basis of a macroscopic model of inertial one-phase flow in homogeneous porous media (solid phase σ, fluid phase β) proposed in [3] and given by

$$\nabla \cdot \langle \mathbf{v}_\beta \rangle = 0 \tag{1}$$

$$\langle \mathbf{v}_\beta \rangle = \frac{-\mathbf{K}}{\mu_\beta} \cdot \left(\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g} \right) - \mathbf{F} \cdot \langle \mathbf{v}_\beta \rangle \tag{2}$$

$\langle \psi_\beta \rangle$ and $\langle \psi_\beta \rangle^\beta$ being the superficial and phase intrinsic averages of any quantity ψ associated to the fluid phase β. In equation 2, $\mathbf{K}$ (symmetric definite positive) and $\mathbf{F}$ are the intrinsic permeability and inertial correction tensors respectively, the latter having no special symmetry feature as will be shown later. These two tensors can be computed from the solution of an auxiliary -closure- problem defined on the unit cell of the periodic structure. When lengths are made dimensionless by the cell size l, velocity by $l^2 \left| \nabla \langle p_\beta \rangle^\beta \right| / \mu_\beta$ (and pressure by $l \left| \nabla \langle p_\beta \rangle^\beta \right|$) the closure is given by

$$Re\ \mathbf{v}_\beta^* \cdot \nabla \mathbf{M}^* + \nabla \mathbf{m}^* = \mathbf{I} \tag{3}$$

$$\nabla \cdot \mathbf{M}^* = 0 \quad ; \quad \mathbf{M}^* = 0 \ \textit{at the solid-fluid interface} \quad ; \quad \mathbf{m}^* \textit{ and } \mathbf{M}^* \textit{ are periodic} \tag{4}$$

When $Re = 0$, the solution of the above problem provides $\mathbf{K}$ according to $\mathbf{K} = l^2 \mathbf{K}^* = l^2 \langle \mathbf{M}^* \rangle$, while, for any other value of Re, $\mathbf{F}$ is given by $\mathbf{F} = \mathbf{K}^* \cdot \langle \mathbf{M}^* \rangle^{-1} - \mathbf{I}$. Note that the closure problem involves the velocity field at the microscale, which means that the stationary Navier-Stokes problem must be solved over the unit cell prior to the determination of $\mathbf{F}$. Note also that the closure problem has a Navier-Stokes structure so that the same solver can be used for both the flow and closure. In the sequel of the paper, gravity is neglected. The flow and closure problem solutions are sought numerically using an artificial compressibility based algorithm developed with a finite volume scheme on a staggered uniform grid. The convective term is treated with the help of a modified QUICK scheme [4] and the linear system is solved using a stabilized bi-conjugate gradient method.

In order to investigate the deviation to Darcy's law, we focus on the contribution of inertia $\mathbf{f_i} = -\mu_\beta \mathbf{K}^{-1} \cdot \mathbf{F} \cdot \langle \mathbf{v}_\beta \rangle$ to the total force exerted on the structure. For convenience, $\mathbf{f_i}$ can be renormalized by a factor inspired from the contribution of the Darcy part so that the result $\mathbf{f_c}$ is given by $\mathbf{f_c} = \frac{\mathbf{K}}{\mu_\beta |\langle \mathbf{v}_\beta \rangle|} \cdot \mathbf{f_i} = \frac{-\mathbf{F} \cdot \langle \mathbf{v}_\beta \rangle}{|\langle \mathbf{v}_\beta \rangle|}$. In the following, the correction $\mathbf{f_c}$ to Darcy's law and some features of the tensor $\mathbf{F}$ are analyzed in the case of the 2D unit cell represented in figure 1.

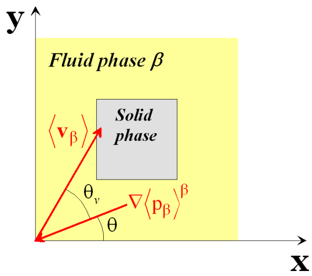

Figure 1: 2D Unit cell used for the inertial correction analysis.

This model structure exhibits an isotropic permeability ($\mathbf{K} = k\mathbf{I}$) and we took the porosity $\varepsilon = 0.75$. Results are presented in terms of a Reynolds number given by $Re_k = \rho_\beta \left| \langle \mathbf{v}_\beta \rangle \right| \sqrt{k} / \mu_\beta$

and a pressure gradient orientation θ defined by the inclination of $\nabla\langle p_\beta\rangle^\beta$ on $\mathbf{e_x}$ for $0 \leq Re_k \leq 10$, $0° \leq \theta \leq 90°$.

2 Results

Results on the four components of the tensor F obtained in the above mentioned configuration are reported in figure 2. As expected, the unit cell symmetries imply $f_{xx}(90° - \theta) = f_{yy}(\theta)$ and $f_{xy}(90° - \theta) = f_{yx}(\theta)$. Clearly, $\mathbf{F}$ is symmetric only when $\theta = 0°$, $\theta = 90°$ and $\theta = 45°$ corresponding to three symmetry axes of the unit cell, while for the latter case, $f_{xx} = f_{yy}$. More generally, when, for an isotropic structure, $\nabla\langle p_\beta\rangle^\beta$ is along a symmetry axis of the unit cell and leads to a flow for which the principal axes of $\mathbf{F}$ are not the periodicity axes $\mathbf{e_x}$ and $\mathbf{e_y}$, then $\mathbf{F}$ is symmetric and $f_{xx} = f_{yy}$. The non-symmetric character of $\mathbf{F}$, as indicated by the difference $f_{xy} - f_{yx}$, increases with Re_k. Values of θ leading to $f_{xy} - f_{yx}$ extrema vary from $\theta \sim 22.5°$ and $\theta \sim 67.5°$ at very low Reynolds numbers (these two values of θ correspond to the angle bissectors of the three symmetry axes) to $\theta \sim 31°$ and $\theta \sim 59°$ at $Re_k \simeq 6.7$ for instance.

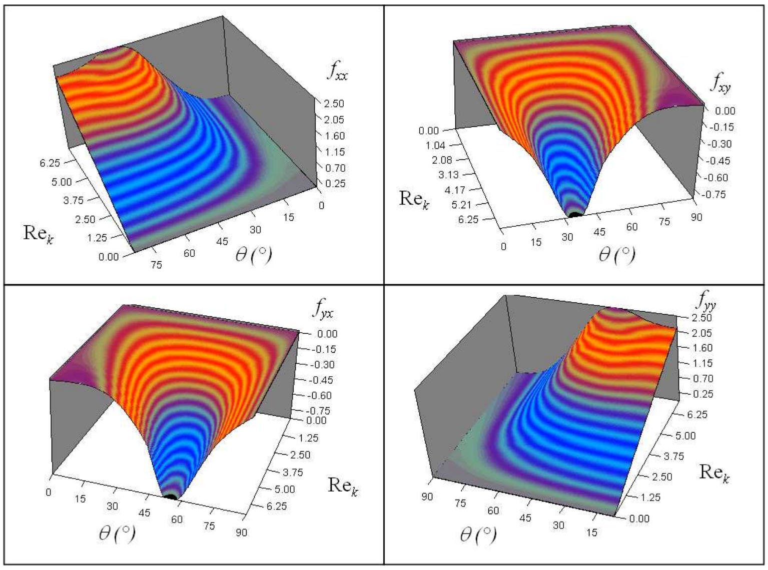

Figure 2: Components of the tensor $\mathbf{F}$ versus Re_k and θ.

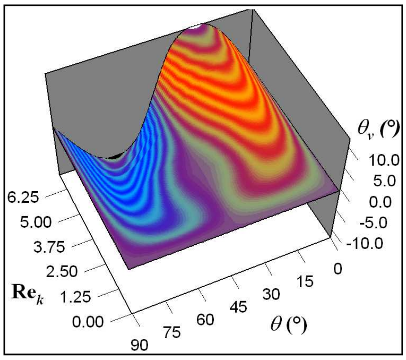

Figure 3: Angle between $\langle \mathbf{v}_\beta\rangle$ and $\nabla\langle p_\beta\rangle^\beta$ versus Re_k and θ.

The absence of symmetry of $\mathbf{F}$ suggests that the average velocity is not aligned with the imposed $\nabla\langle p_\beta\rangle^\beta$ and that the net macroscopic force exerted on the structure is not a pure drag. This is clearly highlighted by the dependence of the angle θ_v, between $\langle \mathbf{v}_\beta\rangle$ and $\nabla\langle p_\beta\rangle^\beta$ as represented in figure 3.

For the same evident symmetry reasons of the unit cell as those mentioned above, the inertial correction is such that , $f_{cy}(\theta) = f_{cx}(90° - \theta)$ and $|\mathbf{f_c}(\theta)| = |\mathbf{f_c}(90° - \theta)|$, so that it is sufficient to inspect f_{cx} for $0 < \theta < 90°$ and $|\mathbf{f_c}|$ for $0 < \theta < 45°$ only. These dependences are reported in figures 4 and 5. For a fixed value of Re_k, f_{cx} and $|\mathbf{f_c}|$ are always minimum when $\theta = 0$. Due to the unit cell symmetry, $|\mathbf{f_c}|$ is maximum for $\theta = 45°$ whatever Re_k. However, the value of θ for which f_{cx} is maximum depends on Re_k. This behavior indicates that conclusions on the correction due to inertia might be incomplete when extracted from a result on its modulus only.

A detailed analysis of f_{cx} and $|\mathbf{f_c}|$ versus Re_k clearly shows that these two quantities are indeed linearly related to Re_k^2 when Re_k is small enough (typically $Re_k < 0.15$ here). This confirms theoretical and numerical results [5, 6] where a weak inertia regime in which the velocity magnitude correction depends on $|\langle \mathbf{v}_\beta\rangle|^3$. However, a linear dependence of f_{cx} and $|\mathbf{f_c}|$

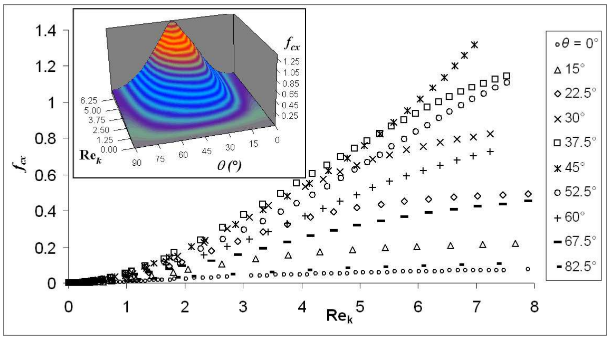

Figure 4: f_{cx} versus Re_k and θ.

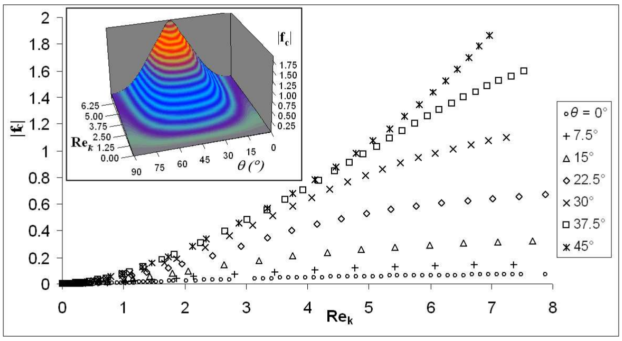

Figure 5: $|\mathbf{f_c}|$ versus Re_k and θ.

on $|\langle \mathbf{v}_\beta \rangle|^2$ corresponding to a strong inertia regime at higher Reynolds numbers is an approximation for any value of θ. A singular behavior even exists when $\theta = 45°$. For this value of θ, a quadratic dependence of both f_{cx} and $|\mathbf{f_c}|$ on Re_k is revealed over the whole range of Re_k investigated here ($Re_k \leqslant 7$).

3 Conclusion

For well ordered structures, the analysis of the inertial correction to Darcy's law shows that, in the general case: i) this correction involves a tensor that is not symmetric even for an isotropic structure from the point of view of its permeability. This tensor is symmetric only when the pressure gradient at the macroscopic scale is along a symmetry axis of the periodic unit cell; ii) the vector of correction is not aligned with the applied pressure gradient implying that the net macroscopic force exerted on the structure is not a pure drag; iii) onset of the deviation to Darcy's law always occurs in a weak inertia regime corresponding to a correction varying with the cube of the velocity; iv) the quadratic correction, as in the classical Forchheimer's law, is an approximation that does not exist for certain particular orientations of the pressure gradient.

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